Year 12 Intro 3

Question 1

Skill involved: 413a: Complete the square for quadratic expressions given in the form $\mathbf{x^2 + bx + c}$ where b is even.

Express $x^2 + 10x - 15$ in the form $(x + p)^2 + q$.

.....

(2 marks)

Question 2

Skill involved: 413a: Complete the square for quadratic expressions given in the form $\mathbf{x^2 + bx + c}$ where b is even.

Express $x^2 - 6x - 81$ in the form $(x - p)^2 + q$.

.....

(2 marks)

Question 3

Skill involved: 414b: Complete the square for quadratic expressions given in the form $\mathbf{ax^2 + bx + c}$, where a is negative.

Express $7 - 4x - x^2$ in the form $p - (x + q)^2$ where p and q are constants.

$p = \ldots\ldots\ldots, q = \ldots\ldots\ldots$

(2 marks)

Question 4

Skill involved: 414a: Complete the square for quadratic expressions given in the form $\mathbf{ax^2 + bx + c}$, where a is positive.

Write $2x^2 - 8x + 9$ in the form $a(x + b)^2 + c$.

.....

(3 marks)

Question 5

Skill involved: 414a: Complete the square for quadratic expressions given in the form $\mathbf{ax^2 + bx + c}$, where a is positive.

Write $3x^2 - 12x + 7$ in the form $a(x + b)^2 + c$

.....

(3 marks)

Question 6

Skill involved: 367d: Solve quadratic equations given in the form $\mathbf{ax^2 \pm bx \pm c = 0}$, requiring factorising.

Solve by factorisation.

$2x^2 + 5x - 12 = 0$

$x =$.....

or $x =$.....

(3 marks)

Question 7

Skill involved: 367b: Solve quadratic equations given in the form $\mathbf{x^2 \pm bx \pm c = 0}$, solvable by factorisation.

Solve by factorisation.

$3x^2 + 11x - 20 = 0$

$x =$.....

or $x =$.....

(3 marks)

Question 8

Skill involved: 417b: Solve a quadratic equation using the quadratic formula to get numeric solutions.

Solve the equation $3x^2 + 9x - 2 = 0$.

Give your answers correct to 1 decimal place.

$x =$.....

or $x =$.....

(3 marks)

Question 9

Skill involved: E417: Quadratic formula to solve quadratic equations

Solve the equation

$10x^2 + 3x - 11 = 0$

Give your answers correct to 1 decimal place.

$x =$.....

or $x =$.....

Question 10

Skill involved: 462d: Solve an equation requiring addition or subtraction of algebraic fractions, leading to a quadratic equation.

Solve

$\frac{3}{x-2} + \frac{2}{x-1} = 5$

Do **not** use trial and improvement.
Write your solutions to 3 significant figures.

$x =$.....

or $x =$.....

(6 marks)

Question 11

Skill involved: 415b: Solve a quadratic equation by completing the square.

Solve $x^2 + 8x + 6 = 0$ by completing the square.
Give your answer in the form $a \pm \sqrt{b}$, where a and b are integers.

$x =$.....

or $x =$.....

(4 marks)

Question 12

Skill involved: 417a: Solve a quadratic equation to get exact solutions.

Solve $x^2 + 6x + 2 = 0$

Give your answer in the form $a \pm \sqrt{b}$ where a and b are integers.

$x =$.....

or $x =$.....

(4 marks)

Question 13

Skill involved: 492g: Use the discriminant to determine the values for algebraic coefficients in a quadratic function, where there are no roots or distinct roots.

Determine the nature of the roots of the function $f(x) = 7x^2 + 5x - 1$.

.....

(2 marks)

Question 14

Skill involved: E492: Discriminant of a quadratic function

Determine the nature of the roots of the equation

$3x^2 - 2x + 7 = 0$

"two real and distinct roots" [ ]

"one real repeated root" / "equal roots" [ ]

"no real roots" [ ]

Question 15

Skill involved: E492: Discriminant of a quadratic function

Find the value of q for which the equation

$qx^2 - 6x + 18 = 0$

has one repeated real root.

$q =$ .....

Question 16

Skill involved: 368g: Determine the x- and y-intercepts of a quadratic graph with equation given in the form $\mathbf{y = ax^2 + bx + c}$

A curve has the equation $y = 2x^2 - 8x - 5$.

Find the coordinates of the point where the curve intercepts the y-axis.

(..... , ...)

(1 mark)

Question 17

Skill involved: E368: Quadratic graphs and their features

Below is a sketch of the graph with equation $y = x^2 - 3x - 18$.

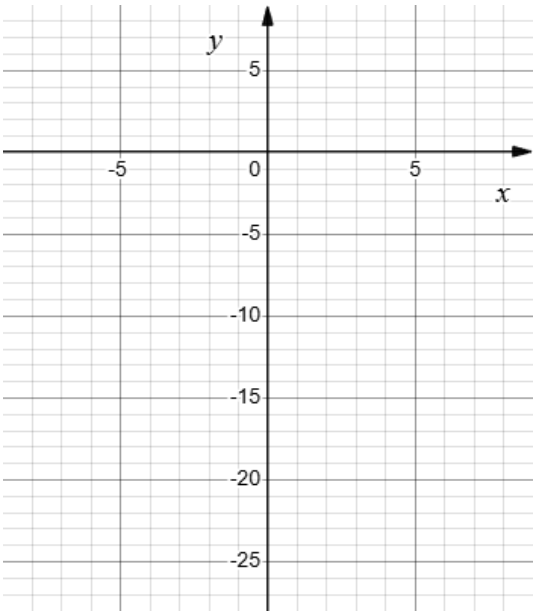
Work out the values of a and b.

$a = \ldots\ldots\ldots, b = \ldots\ldots\ldots$

Question 18

Skill involved: 368t: Determine the coordinate of the turning point of a quadratic using symmetry and a sketch of the graph.

Sketch the graph of $y = (x - 6)(x + 4)$.
On your sketch, show clearly the points of intersection with the x-axis and the y-axis, and the coordinates of the turning point.

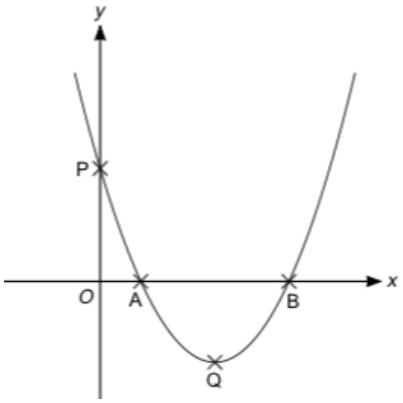


(2 marks)

Question 19

Skill involved: 368t: Determine the coordinate of the turning point of a quadratic using symmetry and a sketch of the graph.

This is a sketch of the graph of $y = (x - 1)(x - 3)$.



Work out the coordinates of the turning point Q.

(..... , ...)

(3 marks)

Question 20

Skill involved: 416a: Determine the minimum point of a quadratic graph when given in completed square form.

You are given that $x^2 + 6x + 2 \equiv (x + 3)^2 - 7$

Write down the coordinates of the minimum point on the curve $y = x^2 + 6x + 2$

(..... , ...)

(1 mark)

Question 21

Skill involved: 416c: Determine the maximum point of a quadratic graph when given in completed square form.

The curve C has equation $y = 3 - 5(x + 1)^2$
The point A is the maximum point on C.

Write down the coordinates of A.

(..... , ...)

(1 mark)

Question 22

Skill involved: 416d: Determine the minimum point of a quadratic graph.

By completing the square, find the coordinates of the turning point of the curve with equation $y = x^2 + 10x + 18$.

(..... , ...)

(3 marks)

Mark scheme

Question 1

$(x+5)^2-40$

•1 correct bracket with square	•1 $(x+5)^2\ldots$
•2 complete process	•2 $(x\ldots5)^2-40$

Question 2

$(x-3)^2-90$

•1 correct bracket with square	•1 $(x-3)^2\ldots$
•2 complete process	•2 $(x-3)^2-90$

Question 3

$p=11, q=2$

$11-(x+2)^2$	2	M1	For $11-(x+q)^2$ or $p-(x+2)^2$
		A1	fully correct, accept $p=11$, $q=2$

Question 4

$2(x-2)^2+1$

$2(x^2-4x)+9$ **or** $2(x^2-4x+\frac{9}{2})$			AO1	M1
$2((x-2)^2-2^2)+9$ **or** $2((x-2)^2-2^2+\frac{9}{2})$				M1
	$2(x-2)^2+1$	3		A1

Question 5

$3(x-2)^2-5$

$3(x^2-4x)+7$ or $3\left(x^2-4x+\frac{7}{3}\right)$	$3(x-2)^2-5$	3	M1	or expanding $a(x^2+2bx+b^2)+c$
$3((x-2)^2-4)+7$ or $3\left((x-2)^2-4+\frac{7}{3}\right)$ or $3(x-2)^2-12+7$			M1	$-12=2ab$ or $7=ab^2+c$
			A1	or $a=3$, $b=-2$, $c=-5$

Question 6

$x = -4$ or $x = 1.5$

$(2x - 3)(x + 4)$ oe 1.5 oe and −4	2 1	**M1** for any two factors that give two correct terms when expanded Correct or **FT** *their* two factors	If they use another method then award **B1** for both answers correct.

Question 7

$x = \frac{4}{3}$ or $x = -5$

$(3x - 4)(x + 5)$ and $\frac{4}{3}$ oe and −5	3	**B2** for $(3x - 4)(x + 5)$ or **B1** for two factors giving two correct terms and **B1FT** for two answers correct from *their* factors If **0** scored then **B1** for two correct answers

Question 8

$x = -3.2$ or $x = 0.2$

•1 correct substitution into quadratic formula	•1 $\frac{-9 \pm \sqrt{9^2 - 4 \times 3 \times (-2)}}{2 \times 3}$
•2 evaluate discriminant	•2 105 (stated or implied by •3)
•3 calculate both roots correct to one decimal place	•3 −3·2, 0·2

Note that the following marking instruction was also given for this question:

1. Correct answer without working award 0/3

Question 9

$x = 0.9$ or $x = -1.2$

This cannot be solved by factorising. When a question asks for answers to be rounded, it is an indication that the quadratic formula should be used:

$10x^2 + 3x - 11 = 0$

Note that

$a = 10$

$b = 3$

$c = -11$

Write down the quadratic formula:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Substitute the values of a, b and c:

$x = \frac{-3 \pm \sqrt{3^2 - 4(10)(-11)}}{2(10)}$

$x = \frac{-3 \pm \sqrt{9 + 440}}{20}$

$x = \frac{-3 \pm \sqrt{449}}{20}$

Separate the two solutions:

$x = \frac{-3 + \sqrt{449}}{20} = 0.909 \ldots$

or $x = \frac{-3 - \sqrt{449}}{20} = -1.209 \ldots$

$\therefore \quad x \approx 0.9 \quad \text{or} \quad x \approx -1.2 \quad (\text{to 1 d. p})$

Question 10

$x = 1.23$ or $x = 2.77$

$3(x-1)$ or $3x-3$ or $2(x-2)$ or $2x-4$	M1	
$3x-3$ and $2x-4$ or $5x-7$	M1	Implies M1 M1
$5(x-1)(x-2)$ or $5(x^2-2x-x+2)$ or $5x^2-15x+10$ or $(x-1)(x-2)$ expanded and multiplied by 5	M1	oe Allow one error in four term expansion of $5(x-1)(x-2)$ Implied by $5(x^2-3x+k)$ or $5(ax^2-3x+2)$
$5x^2-20x+17\ (=0)$	M1dep	dep on 3rd M1 oe 3-term quadratic equation eg $5x^2-20x=-17$ Correctly collects terms in their expansion
$\frac{--20\pm\sqrt{(-20)^2-4\times5\times17}}{2\times5}$ or $\frac{10\pm\sqrt{15}}{5}$ or $(x-2)^2-4=-\frac{17}{5}$ or $5[(x-2)^2-4]=-17$	M1	oe Correct use of quadratic formula for their 3-term quadratic eg $(-20)^2$ can be 20^2 or correct factorisation of their 3-term quadratic or attempt to complete the square for their 3-term quadratic Must be correct up to form $(x-a)^2+b=c$ or $k[(x-d)^2+e]=f$
1.23 and 2.77	A1	Must be 3 significant figures

Question 11

$x = -4+\sqrt{10}$ or $x = -4-\sqrt{10}$

$(x+4)$	M1	Must be in brackets
$(x+4)^2-16+6$	A1	oe
$x+4=\pm\sqrt{\text{their } 10}$	M1dep	Allow their 10 to be negative
$(x=)\ -4\pm\sqrt{\text{their } 10}$	A1ft	ft on one arithmetic error, but only if their 10 is positive $-4+\sqrt{10}$ is 3 marks Correct answer with no working is 3 marks maximum

Question 12

$x = -3+\sqrt{7}$ or $x = -3-\sqrt{7}$

Alternative Method 1

$(x \pm 3)^2 \pm 9$ or ± 7 or ± 11 $(= 0)$	M1	
$(x \pm 3)^2 = 7$ or 11	M1dep	
$x + 3 = \pm\sqrt{7}$	A1	
$-3 \pm \sqrt{7}$	A1ft	ft on one error, ie $3 \pm \sqrt{7}$ or $-3 \pm \sqrt{11}$ SC3 $-3 + \sqrt{7}$

Alternative Method 2

$(x =) \frac{-6 \pm \sqrt{6^2 - 4\times1\times2}}{2}$	M1	Allow one sign error but not partial division or wrong formula
$(x =) \frac{-6 \pm \sqrt{6^2 - 4\times1\times2}}{2}$	A1	No errors
$(x =) \frac{-6 \pm \sqrt{28}}{2}$ or $\frac{6 \pm \sqrt{28}}{2}$ or $\frac{-6 \pm \sqrt{44}}{2}$	M1dep	
$-3 \pm \sqrt{7}$	A1ft	ft on one error, ie $3 \pm \sqrt{7}$ or $-3 \pm \sqrt{11}$ SC3 $-3 + \sqrt{7}$

Question 13

real AND [distinct OR different]

Ans: real and distinct	
•¹ find discriminant	•¹ 53 $\left[5^2 - 4\times7\times(-1)\right]$
•² state nature of roots	•² real and distinct (or equivalent)

Question 14

"no real roots"

To determine the nature of the roots, calculate the "discriminant":

$b^2 - 4ac$

Then:

If $b^2 - 4ac > 0$ then there are "two real and distinct roots".

If $b^2 - 4ac = 0$ then there is "one real repeated root" / "equal roots".

If $b^2 - 4ac < 0$ then there are "no real roots".

Question 15

$\frac{1}{2}$

If a quadratic equation has one repeated real root (also termed "equal roots"), the discriminant must be equal to zero.

Create an equation with $b^2 - 4ac$ equal to zero:

$b^2 - 4ac = 0$

Compare $qx^2 - 6x + 18$ with $ax^2 + bx + c$ to identify the values of a, b and c:

$a = q; \quad b = -6; \quad c = 18$

Substitute the values of a, b and c into the inequation and solve for k:

$b^2 - 4ac = 0$

$(-6)^2 - 4(q)(18) = 0$

$36 - 72q = 0$

$36 = 72q$

$\frac{36}{72} = q$

$q = \frac{1}{2}$

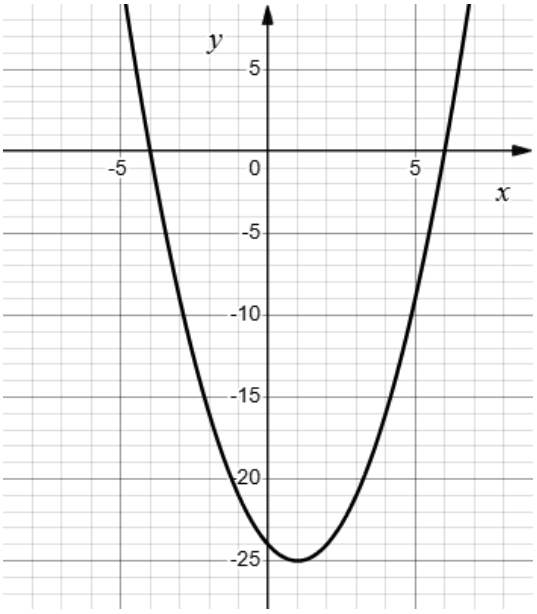
Question 16

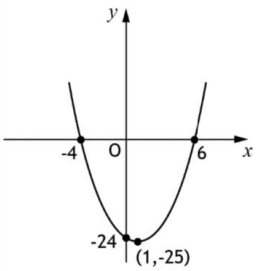
$(0, -5)$

Question 17

$a = -3, b = 6$

Question 18



•[1] identify roots	•[1] −4 **AND** 6
•[2] identify turning point **OR** y-intercept	•[2] (1, −25) **OR** −24
•[3] identify the turning point **AND** the y-intercept and sketch a consistently annotated parabola	•[3] (1, −25) **AND** −24 and consistently annotated parabola (see Note 1). 

Question 19

$(2, -1)$

(2, -1)	**3** 2AO1.3a 1AO2.1a	**M1** for $x = 2$ (allow FT from **(a)**) **M1** for attempt to substitute *their* 2 into $(x - 1)(x - 3)$

Question 20

$(-3, -7)$

(−3, −7)	B1 ft	ft their h and k from part (a) only if $h \neq 0$ and $k \neq 0$

Question 21

$(-1, 3)$

$(-1, 3)$	1	B1	cao

Question 22

$(-5, -7)$

$(-5, -7)$	M1	method to start to complete the square, e.g. $(x + 5)^2$
	M1	$(x + 5)^2 - 7$
	A1	cao (dep on method seen)