Year 12 Intro 2

Question 1

Skill involved: 394f: Raise a number to the power of a non-unit fraction.

Evaluate

$8^{\frac{5}{3}}$

.....

(2 marks)

Question 2

Skill involved: 394r: Write a surd expression in one variable as a power of that variable (or value).

Express $\frac{1}{\sqrt[3]{x}}$ in the form x^n.

.....

(2 marks)

Question 3

Skill involved: 298b: Raise a fraction to a negative integer power.

Find the value of $\left(\frac{1}{5}\right)^{-3}$.

.....

(2 marks)

Question 4

Skill involved: 394r: Write a surd expression in one variable as a power of that variable (or value).

One of the following is equal to $\sqrt{9^{16x^2}}$ for all values of x. Which one?

3^{4x} [ ]

3^{4x^2} [ ]

3^{8x^2} [ ]

9^{4x} [ ]

9^{8x^2} [ ]

Question 5

Skill involved: 394r: Write a surd expression in one variable as a power of that variable (or value).

Simplify $\sqrt{x^5 \times x^9}$

Give your answer in the form x^p where p is an integer.

.....

(2 marks)

Question 6

Skill involved: 394j: Raise a number to the power of a negative fraction.

Which of these has the smallest value?

2016^{-1} [ ]

$2016^{-1/2}$ [ ]

2016^{0} [ ]

$2016^{1/2}$ [ ]

2016^{1} [ ]

Question 7

Skill involved: 194l: Simplify an expression using multiple index laws with multiple algebraic variables.

$\frac{(a^r)^2}{(a^t)^3}$

can be written in the form a^u

Find an expression for u in terms of r and t.

$u =$ .....

(2 marks)

Question 8

Skill involved: 158d: Use laws of indices for both multiplying and dividing powers.

$\left(13^{-6}\right)^4 \times 13^5 = 13^k$

Find the value of k.

$k = \ldots\ldots\ldots$

(2 marks)

Question 9

Skill involved: 394r: Write a surd expression in one variable as a power of that variable (or value).

Write $\sqrt{\frac{1}{m^6}}$ as a single power of m.

.....

(2 marks)

Question 10

Skill involved: 394r: Write a surd expression in one variable as a power of that variable (or value).

Write $\frac{\sqrt[3]{81}}{3}$ in the form 3^k, where k is a constant to be found.

$k = \ldots\ldots\ldots$

(3 marks)

Question 11

Skill involved: 358d: Solve equations by transforming to the same base, leading to a linear equation.

Given that $3^x = 9^{x+1}$ work out the value of x.

$x = \ldots\ldots\ldots$

(2 marks)

Question 12

Skill involved: 358d: Solve equations by transforming to the same base, leading to a linear equation.

$128 = 4^{2x} \times 2^x$

Work out the value of x.

$x = \ldots\ldots\ldots$

(3 marks)

Question 13

Skill involved: E334: Simplifying surds

Simplify $\sqrt{20}$.

.....

Question 14

Skill involved: 334a: Simplify a surd.

Write $\sqrt{50}$ in the form $k\sqrt{2}$, where k is an integer.

.....

Question 15

Skill involved: E334: Simplifying surds

Simplify $\sqrt{x^6\, y^{10}}$, giving your answer in terms of x and y.

.....

Question 16

Skill involved: E334: Simplifying surds

Fully simplify $6\sqrt{2} + 2\sqrt{18}$.

.....

Question 17

Skill involved: 337c: Expand two brackets involving surds given in the form $(\mathbf{a} + \sqrt{\mathbf{b}})(\mathbf{c} + \sqrt{\mathbf{b}})$

(b) Expand $(2 + \sqrt{3})(1 + \sqrt{3})$

Give your answer in the form $a + b\sqrt{3}$, where a and b are integers.

.....

(2 marks)

Question 18

Skill involved: 337c: Expand two brackets involving surds given in the form $(\mathbf{a}+\sqrt{\mathbf{b}})(\mathbf{c}+\sqrt{\mathbf{b}})$

Work out

$(2+\sqrt{3})(2-\sqrt{3})$

Give your answer in its simplest form.

.....

(2 marks)

Question 19

Skill involved: 337e: Expand two brackets involving multiples of surds.

Expand $(5+3\sqrt{2})^2$

Give your answer in the form $(a+b\sqrt{2})$, where a and b are integers.

.....

(2 marks)

Question 20

Skill involved: 337g: Multiply three brackets involving surds.

Expand and simplify

$(\sqrt{5}+3)(\sqrt{5}-2)(\sqrt{5}+1)$

.....

(4 marks)

Question 21

Skill involved: 392c: Rationalise the denominator of a fraction given in the form $\frac{\mathbf{a}}{\sqrt{\mathbf{b}}}$ and where subsequent simplification is required.

Rationalise the denominator of $\frac{10}{\sqrt{5}}$

.....

(2 marks)

Question 22

Skill involved: 392h: Simplify $\left(\frac{\mathbf{a}}{\sqrt{\mathbf{b}}}\right)^{\mathbf{n}}$ where $\mathbf{n}$ is large.

Express $\left(\frac{1}{\sqrt{3}}\right)^7$ in the form $\frac{\sqrt{b}}{c}$ where b and c are integers.

.....

(3 marks)

Question 23

Skill involved: E392: Rationalising the denominator where the denominator consists of a single surd term

Evaluate:

$\frac{2}{\sqrt{2}} - \sqrt{2}$

.....

Question 24

Skill involved: 392a: Rationalise the denominator of a fraction given in the form $\frac{\mathbf{a}}{\sqrt{\mathbf{b}}}$

Write in the form $b\sqrt{c}$.

$\frac{14}{\sqrt{7}}$

.....

Question 25

Skill involved: E393: Rationalising the denominator where the denominator consists of a surd and a second term

Rationalise the denominator.

$\frac{\sqrt{2}}{1-\sqrt{3}}$

.....

Question 26

Skill involved: E393: Rationalising the denominator where the denominator consists of a surd and a second term

Rationalise the denominator

$\frac{5-\sqrt{3}}{1+\sqrt{3}}$

.....

Question 27

Skill involved: 393g: Rationalise the denominator of a fraction given in the form $\frac{a+b\sqrt{c}}{d+e\sqrt{c}}$

Rationalise the denominator and simplify

$\frac{5\sqrt{5}-2}{2\sqrt{5}-3}$

.....

(4 marks)

Mark scheme

Question 1

32

Ans: 32	
•[1] interpret index	•[1] $\sqrt[3]{8^5}$
•[2] complete evaluation	•[2] 32

When dealing with a fractional index, it may help to remember the phrase, "the Power is at the top, the Root is at the bottom".

In general,

$x^{\frac{m}{n}} = \sqrt[n]{x}^m$

In this case,

$8^{\frac{5}{3}} = \sqrt[3]{8}^5$

$= 2^5$

$= 32$

Question 2

$x^{-\frac{1}{3}}$

Ans: $x^{-\frac{1}{3}}$	
•[1] apply $\sqrt[n]{x^m} = x^{\frac{m}{n}}$	•[1] $\frac{1}{x^{\frac{1}{3}}}$ stated or implied by •[2]
•[2] apply $\frac{1}{x^n} = x^{-n}$	•[2] $x^{-\frac{1}{3}}$

Question 3

125

14. (a) 125	B2	B1 for sight of 5^3 or for $\left(\frac{1}{125}\right)^{-1}$; allow for sight of $\frac{1}{5^3} = \frac{1}{125}$

Question 4

9^{8x^2}

We use the fact that $\sqrt{a} = a^{\frac{1}{2}}$ when $a > 0$, and the index rule $(a^b)^c = a^{bc}$. It follows that $\sqrt{9^{16x^2}} = (9^{16x^2})^{\frac{1}{2}} = 9^{(16x^2 \times \frac{1}{2})} = 9^{8x^2}$.

To complete the solution we need to show that none of the other expressions is equal to $\sqrt{9^{16x^2}}$ for all values of x. As in the solution to Question 8, it is good enough to find a single value of x for which expressions A, B, C and D, do not have the same value as $\sqrt{9^{16x^2}}$ When $x = 1$, $\sqrt{9^{16x^2}} = 9^8$ while the values of A, B, C and D are 3^4, 3^4, 3^8 and 9^4, respectively. This shows that none of them is equal to $\sqrt{9^{16x^2}}$ for all values of x.

Question 5

x^7

x^7	B2	B1 $\sqrt{x^{14}}$ or $(x^{14})^{\frac{1}{2}}$ or $\sqrt{x^{5+9}}$ or $(x^{5+9})^{\frac{1}{2}}$ or $x^{\frac{14}{2}}$ or $x^{\frac{5+9}{2}}$ or $x^{\frac{5}{2}} \times x^{\frac{9}{2}}$ or $x^{2.5} \times x^{4.5}$	

Question 6

2016^{-1}

Question 7

$2r - 3t$

$2r - 3t$	M1	for a^{2r} or a^{3t}	
	A1	$2r - 3t$ oe	

Question 8

$k = -19$

$13^{-24} \times 13^5$		2	M1	for 13^{-24} or for $k = -6 \times 4 + 5$
	-19		A1	Accept 13^{-19}

Question 9

m^{-3}

m^{-3}	B2	B1 $\sqrt{m^{-6}}$ or $\frac{1}{m^3}$	

Question 10

$k = \frac{1}{3}$

$\frac{\sqrt[3]{81}}{3} = \frac{\sqrt[3]{3^4}}{3}$ or $\frac{\sqrt[3]{81}}{3} = \frac{\sqrt[3]{3^4}}{3}$ $= \frac{3^{\frac{4}{3}}}{3}$ or $\frac{\sqrt[3]{3^3 \times 3}}{3} = \frac{3\sqrt[3]{3}}{3}$ $\left[= 3^{\frac{4}{3}-1}\right] = 3^{\frac{1}{3}}$ or $\sqrt[3]{3} = 3^{\frac{1}{3}}$	**M1** **M1dep** **A1** **3 AO2.2**	$\frac{\sqrt[3]{81}}{3} = \frac{\sqrt[3]{81}}{\sqrt[3]{3^3}}$ $= \sqrt[3]{\frac{81}{27}}$ $= \sqrt[3]{3} = 3^{\frac{1}{3}}$	In left-hand methods, **M1M1** can be awarded if the denominator 3 is consistently omitted There may be other surd methods. **M1** first productive step $\sqrt[3]{81} = 81^{\frac{1}{3}}$ is not sufficiently productive as a first step **M1dep** second productive step from a correct first step Conversion to decimals scores **0**

Question 11

$x = -2$

$(x =)\ 2(x + 1)$ or $2x + 1$ or $\frac{1}{2}x\ (= x + 1)$	M1	oe May be seen as an index is $(3^2)^{x+1}$ or $9^{1/2x}$
–2	A1	Correct answer is 2 marks even if working nonsense or wrong.

Question 12

$x = 1.4$

$2^7 = 4^{2x} \times 2^x$ or $128 = (2^2)^{2x} \times 2^x$		3	M1	Replacing 128 by 2^7 or 4 by 2^2
$7 = 2(2x) + x$			M1	
	1.4		A1	oe

Question 13

$2\sqrt{5}$

Question 14

$5\sqrt{2}$

Question 15

x^3y^5

Question 16

$12\sqrt{2}$

Question 17

$5 + 3\sqrt{3}$

$2\times1+2\times\sqrt{3}+1\times\sqrt{3}+\sqrt{3}\times\sqrt{3}$	$5+3\sqrt{3}$	2	M1 for $2\times1+2\times\sqrt{3}+1\times\sqrt{3}+\sqrt{3}\times\sqrt{3}$ or three of 2, $2\sqrt{3}$, $\sqrt{3}$, $\sqrt{9}$ (or 3 or $\sqrt{3^2}$ or $\left(\sqrt{3}\right)^2$) A1 for $5+3\sqrt{3}$ cao (SC: B1 for $a+3\sqrt{3}$ or $5+b\sqrt{3}$ if M0 scored, where a and b are integers not equal to 0)

Question 18

1

$(2+\sqrt{3})(2-\sqrt{3})$ $= 4-2\sqrt{3}+2\sqrt{3}-\sqrt{3}\sqrt{3}$ $= 4-3$	1	2	M1 for all 4 terms correct ignoring signs or 3 out of 4 terms with correct signs.or correct use of difference of 2 squares A1 cao (SC M1 for $4-2\sqrt{3}+2\sqrt{3}$)

Question 19

$43 + 30\sqrt{2}$

$25+15\sqrt{2}+15\sqrt{2}+9\times2$ or $25+15\sqrt{2}+15\sqrt{2}+18$ or $25+30\sqrt{2}+9\times2$ or $25+30\sqrt{2}+18$		2	M1	Expand to give four terms – (must have surds not decimals), at least three correct, or three terms with irrational term and one other correct. Accept $\left(\sqrt{2}\right)^2$ for 2
	$43+30\sqrt{2}$		A1	dep on M1 awarded

Question 20

4

Multiplying 1st and 2nd brackets first

$\sqrt{5}\ \sqrt{5}+3\sqrt{5}-2\sqrt{5}-3\times2$	M1	or better
$\sqrt{5}-1$	A1	
$\sqrt{5}\ \sqrt{5}-\sqrt{5}+\sqrt{5}-1$	M1	or better this is $(\sqrt{5}-1)(\sqrt{5}+1)$
4	A1	

Question 21

$2\sqrt{5}$

$\frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} \left(= \frac{10\sqrt{5}}{5}\right)$	M1
$= 2\sqrt{5}$	A1

Question 22

$\frac{\sqrt{3}}{81}$

$\frac{\sqrt{3}}{81}$	M1	for simplifying the power eg $(\sqrt{3})^7 = 27\sqrt{3}$	
	M1	for method to rationalise the denominator eg multiplying by $\frac{\sqrt{3}}{\sqrt{3}}$	May be seen as the first step
	A1	for $\frac{\sqrt{3}}{81}$ **or** equivalent fraction in form $\frac{\sqrt{b}}{c}$, eg $\frac{\sqrt{2187}}{2187}$	

Question 23

0

Question 24

$2\sqrt{7}$

Question 25

$-\frac{\sqrt{6}+\sqrt{2}}{2}$ or $-\frac{\sqrt{6}}{2} - \frac{\sqrt{2}}{2}$

Question 26

$-4 + 3\sqrt{3}$

Question 27

$4 + \sqrt{5}$

$\frac{(5\sqrt{5}-2)(2\sqrt{5}+3)}{(2\sqrt{5}-3)(2\sqrt{5}+3)}$	M1	
$50-4\sqrt{5}+15\sqrt{5}-6$ **or** $20-9$	M1	oe allow one error
$50-4\sqrt{5}+15\sqrt{5}-6$ **and** $20-9$	A1	oe
$4+\sqrt{5}$	A1	